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Anti-de Sitter fragmentation

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Abstract: Low-energy, near-horizon scaling limits of black holes which lead to string theory on $AdS_2 \times S^2$ are described. Unlike the higher-dimensional cases, in the simplest approach all finite-energy excitations of $AdS_2 \times S^2$ are suppressed. Surviving zero-energy configurations are described. These can include tree-like structures in which the $AdS_2 \times S^2$ throat branches as the horizon is approached, as well as disconnected $AdS_2 \times S^2$ universes. In principle, the black hole entropy counts the quantum ground states on the moduli space of such configurations. In a nonsupersymmetric context AdS_D for general D can be unstable against instanton-mediated fragmentation into disconnected universes. Several examples are given.

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1. Introduction

By now a beautiful and coherent story has been developed for the AdS_D/CFT_{D-1} duality [1] for several values of D. A notable exception is the enigmatic case $D = 2$. This case is perhaps the most interesting from the point of view of black hole physics because it is the very-near-horizon geometry of all known cases of supersymmetric black holes with non-zero entropy.[1] One immediately puzzling feature is the fact that AdS_2 has two disconnected boundaries. From this alone it is evident that the $D = 2$ case must involve qualitatively new features. Some preliminary progress on this case was reported in [3, 4, 5, 2, 6].

[1] In some cases one encounters quotients of AdS_3, but these also reduce to AdS_2 at sufficiently low energies.[2]

In this paper we shall continue exploration of the AdS_2/CFT_1 duality. In section 2 we analyze several inequivalent approaches to the near-horizon limit. We shall see that it is *not* possible to keep the charge, energy and temperature fixed in the usual manner while taking the Planck mass $M_p \to \infty$. In the most straightforward near-horizon limit the excitation energy of AdS_2 is forced to zero. The resulting theory describes only the (many) extremal black hole ground states.

AdS_2 is not the only zero-energy configuration which survives the $M_p \to \infty$ limit. In addition one can have geometries which are asymptotic to AdS_2 at large radius but branch (in a tree-like structure) into smaller AdS_2 regions as one moves toward the horizon. In section 3 we describe these configurations and their low energy dynamics. We further discuss their description in the dual CFT.

In section 4 we consider general AdS_D spaces and discuss the possibility of brane creation by the antisymmetric D-form field strength [7, 8]. If $D > 2$, it can only occur in non-supersymmetric cases. We present an example of a non-supersymmetric $AdS_3 \times S^3 \times K3$ compactification where this seems to be the dominant decay mode. Finally we discuss topology changing instantons in the supersymmetric AdS_2 case. These instantons, found by Brill [9], describe tunneling between several AdS_2 spaces.

2. AdS_2 as a low-energy limit

The oldest and simplest example of AdS_2 arising as a near-horizon geometry is in the context of the Reissner-Nordström solution of four-dimensional Einstein-Maxwell gravity. The full magnetically-charged solution is

$$\begin{aligned} ds^2 &= -\frac{(r-r^+)(r-r^-)}{r^2}dt^2 + \frac{r^2}{(r-r^+)(r-r^-)}dr^2 + r^2 d\Omega_2^2\,, \\ F &= Q\epsilon_2\,, \\ r_\pm &= QL_p + EL_p^2 \pm \sqrt{2QEL_p^3 + E^2L_p^4}\,. \end{aligned} \tag{2.1}$$

In this expression L_p is the Planck length ($L_p^2 = G_N$), ϵ_2 is the volume element on the unit S^2 and

$$E = M - \frac{Q}{L_p} \tag{2.2}$$

is the excitation energy above extremality. String theoretic examples generically involve more fields and several charges. We will mainly consider the Reissner-Nordström example because it is simpler and has qualitatively similar behavior.

An important feature of these black holes [10] is that the semiclassical analysis of their thermodynamic behavior breaks down very near extremality. This follows from the formulae for the entropy and Hawking temperature

$$S_{BH} = \frac{\pi r_+^2}{L_p^2}\,, \qquad T_H = \frac{r_+ - r_-}{4\pi r_+^2}\,. \tag{2.3}$$

Near extremality the energy-temperature relation is

$$E \sim 2\pi^2 Q^3 T_H^2 L_p \,. \tag{2.4}$$

The energy of a typical quantum of Hawking emission is of order T_H. When this energy is of order of or greater than the total available energy E above extremality, the semiclassical analysis must break down. This occurs at an excitation energy of order

$$E_{gap} \equiv \frac{1}{Q^3 L_p} \,. \tag{2.5}$$

In string theory examples the nature of this breakdown is well-understood [11] : the black hole has a mass gap and (2.5) is the energy of its lowest-lying excitation.[2] In the description of a four-dimensional black hole given in [12] the gap state is the lowest excitation of a conformal field theory on a circle. In more general stringy constructions of four-dimensional black holes it is not always possible to compute the gap but the semiclassical analysis of [10] indicates it will always be of the order (2.5).

The near-horizon limit is simplest to describe for the extremal case in which $E = 0$, $r_+ = QL_p$ and $T_H = 0$. One then considers the limit

$$L_p \to 0 \,, \tag{2.6}$$

with

$$U = \frac{r - r_+}{L_p^2} \,, \qquad Q \text{ fixed} \,. \tag{2.7}$$

The metric then reduces to

$$\frac{ds^2}{L_p^2} = -\frac{U^2}{Q^2}dt^2 + \frac{Q^2}{U^2}dU^2 + Q^2 d\Omega_2^2 \,. \tag{2.8}$$

In null coordinates

$$u^{\pm} = \arctan\left(t \pm \frac{Q^2}{U}\right) \,, \tag{2.9}$$

the metric is

$$\frac{ds^2}{L_p^2} = -\frac{4Q^2 du^+ du^-}{\sin^2(u^+ - u^-)} + Q^2 d\Omega_2^2 \,. \tag{2.10}$$

This is known as the Robinson-Bertotti geometry on AdS_2 $\times S^2$. As illustrated in figure 1, the $AdS_2 \times S^2$ region of the full Reissner-Nordström geometry is a ribbon which

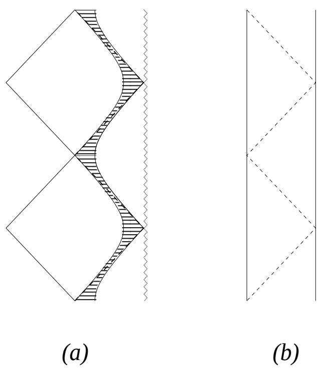

Figure 1: *a*) Penrose diagram corresponding to the extremal Reissner-Nordström black hole. The dashed line is the black hole horizon, and the shaded strip is the near-horizon AdS_2 region. *b*) Penrose diagram for AdS_2. The diagonal lines are the horizons inherited from the embedding in Reissner-Nordström.

[2]There are however lower-energy modes describing the fragmentation of the black hole into smaller pieces. These modes are discussed below.

zigzags its way up through the infinite chain of universes. One of the timelike boundaries of AdS_2 ($u^+ = u^-$) is just outside the black hole horizon, while the other ($u^+ = u^- + \pi$) is just inside.

Since there are two timelike boundaries, quantum gravity with fermions on AdS_2 will have a NS and R sector. In the supersymmetric case, the Hamiltonian H_0 which generates time translations of the full solution (2.1) is the square of a supercharge: $H_0 = Q^2$. This Hamiltonian generates accelerating trajectories in the global coordinates (2.10) of the near-horizon AdS_2. The NS sector has a supercharge ($G_{1/2}+G_{-1/2}$) which squares to H_0, but the R sector does not. Therefore in the supersymmetric case the near-horizon limit leads to the NS sector of quantum gravity on AdS_2.

One may also wish to consider more general limits which are not restricted to zero temperature and excitation energy. This problem is qualitatively different than its higher-dimensional cousins because of the explicit factors of L_p appearing in (2.4) and (2.5). In the following subsections we consider four such more general limits.

2.1 First limit: $L_p \to 0$, $(E,\ Q)$ fixed

The limit $L_p \to 0$ with $(E,\ Q)$ fixed is problematic because according to (2.4) the Hawking temperature is infinite and the geometry is singular. Hence we do not know how to make sense of this limit. Note that for higher ($p > 0$) p-branes with AdS horizons the energy-temperature relation extracted from the near-extremal solutions is of the form

$$E \sim V_p T_H^{p+1}\,. \tag{2.11}$$

This involves no explicit factors of L_p (by dimensional analysis) and the $L_p \to 0$ limit is nonsingular for either fixed temperature or energy. In contrast the energy-temperature relation (2.5) is characteristic of a string-like rather than a point-like object. In place of the missing brane dimension a power of L_p appears in (2.4).

2.2 Second limit: $L_p \to 0$, $(T_H,\ Q)$ fixed

In order to keep the geometry of the solution fixed as $L_p \to 0$, one should keep the temperature T_H, which is related to the periodicity of the solution, fixed. From (2.4) it immediately follows that the excitation energy E vanishes in such a limit, while the gap energy (2.5) goes to infinity. Defining U exactly as in (2.7) one finds that the metric reduces to

$$\frac{ds^2}{L_p^2} = -\frac{U(U+4\pi Q^2 T_H)}{Q^2}dt^2 + \frac{Q^2}{U(U+4\pi Q^2 T_H)}dU^2 + Q^2 d\Omega_2^2\,. \tag{2.12}$$

The metric (2.12) takes the canonical form (2.8) in the primed coordinates

$$t' \pm \frac{Q^2}{U'} = \tanh\left[\pi T_H\left(t \pm \frac{1}{4\pi T_H}\ln\frac{U}{U+4\pi Q^2 T_H}\right)\right]. \tag{2.13}$$

Hence at the classical level the geometry of the $L_p \to 0$ limit is independent of T_H.

At the quantum level there is a distinction. The quantum vacuum depends on the choice of time coordinate used to distinguish positive and negative frequency oscillators. The $SL(2,\mathbb{R})$ invariant vacuum leads to a thermal bath of particles in the coordinates (2.12). The energy density of this thermal bath for the case of a conformal matter system with central charge c is

$$T_{00} = \frac{c\pi T_H^2}{6}\,. \tag{2.14}$$

The impossibility of non-singular finite energy excitations of $AdS_2 \times S^2$ at the classical level can be seen directly from the classical equations. The classical action in two dimensions contains the gravity-dilaton terms

$$S_2 = \frac{1}{4}\int d^2x\sqrt{-g}\left\{e^{-2\phi}\left[R - F^2 + 2(\nabla\phi)^2\right] + \frac{2}{L_p^2}\right\} + S_M\,, \tag{2.15}$$

where S_M is the matter action, R is the two dimensional scalar curvature, $4\pi e^{-2\phi}$ is the volume of the S^2 and $F = dA$ is the gauge field strength. The ++ constraint equation can be written

$$-2e^{-\phi}\nabla_+\nabla_+e^{-\phi} = T_{++} \geq 0\,. \tag{2.16}$$

Integrating (2.16) in conformal gauge $ds^2 = e^{2\rho}du^+du^-$ with the measure $e^{\phi-2\rho}du^+$ across AdS_2 from 0 to π along the line $u^- = 0$ gives

$$e^{-2\rho}\partial_+e^{-\phi}|_{u^+=0} - e^{-2\rho}\partial_+e^{-\phi}|_{u^+=\pi} = \frac{1}{2}\int du^+e^{\phi-2\rho}T_{++} \geq 0 \tag{2.17}$$

and $e^{-2\rho}$ vanishes quadratically near the boundaries for AdS_2. If T_{++} is nonzero, then (2.17) implies that $e^{-\phi}$ must diverge linearly near at least one of the two boundaries.[3] Hence the geometry cannot be asymptotic to $AdS_2 \times S^2$ when T_{++} is nonzero.

This zero-energy constraint might be modified at the quantum level in order to account for the energy of the Hawking radiation. Indeed the quantum constraints contain a one-loop correction from c massless matter fields of the form $\frac{c}{12}(\partial_+^2\rho - (\partial_+\rho)^2)$ which modifies the preceding analysis.

The classical argument does not eliminate the possibility of black-hole-like spacetimes which have non-singular spacelike slices, but contain spacelike singularities in the past and future. For such spacetimes there may not be null surfaces which cross from one boundary to the other. A similar analysis using the time-time constraint equations on spacelike slices may yield relevant information, but one must consider the possibility of negative T_{00} from tachyons.

Although the energy is classically constrained to vanish for nonsingular spacetimes, this limit is far from trivial because it should retain all the ground states of the

[3] The rate of divergence depends on the coefficients in (2.15) which vary among different examples.

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system. The ground state entropy is $S = \pi Q^2$ for large Q. We have not succeeded in understanding the proper description of all these states in this framework. One possibility, discussed below, is that they arise from modes corresponding to black hole fragmentation or the separation of $AdS_2 \times S^2$ universes.

2.3 Third limit: $L_p \to 0$, $(E,\ T_H)$ fixed, $Q \to \infty$

In [3] it was found that Green functions in the near-horizon $AdS_2 \times S^2$ geometry of the Reissner-Nordström black hole agreed with those of a $1+1$ chiral conformal field theory. The detailed agreement persisted when angular momentum was added to the black hole [3] and also for general charges [4] . Since Green functions measure correlations of finite-energy disturbances these results suggest the existence of an AdS_2/CFT_1 correspondence involving non-zero excitation energies. This may seem to contradict the analysis of the previous subsection. However a somewhat different limit was implicit in [3]. One can hold both E and T_H, as well as E_{gap}, fixed as $L_p \to 0$ at the price of taking the charge $Q \sim L_p^{-1/3} \to \infty$. Since Q is diverging this is a large N limit. Defining

$$V = \frac{r - r_+}{Q^2 L_p^2}\,, \tag{2.18}$$

the metric (2.1) in the limit $L_p \to 0$ with fixed T_H, E and E_{gap} defined in (2.5) takes the $AdS_2 \times S^2$ form

$$\frac{(E_{gap}L_p)^{2/3}}{L_p^2}ds^2 = -V(V + 4\pi T_H)dt^2 + \frac{1}{V(V+4\pi T_H)}dV^2 + d\Omega_2^2\,. \tag{2.19}$$

The problem of small energies leading to divergent ϕ does not appear because $e^{-2\phi}$, which appears on the left hand side of (2.16), is of order Q^2 and diverges for $L_p \to 0$. The right hand side is kept finite of order E and can be neglected in comparison. The back reaction of matter on ϕ is suppressed. In string theory one also finds that the massive string modes decouple in this limit. Hence the limit largely consists of the free supergravity on $AdS_2 \times S^2$. Nevertheless as seen in [3] an AdS/CFT duality already has nontrivial content within this limit.

2.4 Fourth limit: L_p small, $(E,\ T_H,\ Q)$ fixed

Let us consider an infrared cutoff $U/Q^2 < \Lambda$ on $AdS_2 \times S^2$. In the dual CFT description this should correspond to an ultraviolet cutoff proportional to Λ. The cutoff theory should be capable of describing states with energies $E \ll \Lambda$. It follows from the discussion in section 2.2 that the addition of energy to $AdS_2 \times S^2$ produces a dilaton which grows like EUL_p^2/Q^4 for large U and small E. So if E is small we can choose a cutoff which satisfies $E \ll \Lambda \ll Q/L_p$ so that the dilaton is small for all $U/Q^2 < \Lambda$. Hence there should be a potentially useful approximate duality relating the cutoff theories at very low energies.

3. AdS_2 trees

The $AdS_2 \times S^2$ geometry (2.8) is not the unique classical charge Q, $E = 0$ configuration which survives the $L_p \to 0$ limit with fixed Q. In addition there are classical solutions corresponding to BPS-saturated multi-black hole solutions with separations of order L_p^2 which survive as distinct objects in the limit $L_p \to 0$. These "tree" geometries are asymptotically $AdS_2 \times S^2$ at large radius, but as one moves inward the geometry branches into smaller $AdS_2 \times S^2$ regions. In a supersymmetric theory the quantum ground states are cohomology classes on the moduli space of such solutions. In principle one might understand the extremal black hole entropy by counting cohomology classes on this space. In this section we will discuss these configurations.

3.1 Two-black hole configurations

Let us begin with the asymptotically flat solution describing two Reissner-Nordström black holes (see fig. 2)

$$\begin{aligned} ds^2 &= -V^{-2}dt^2 + V^2 d\vec{x}^2 \,, \\ *F &= \frac{1}{L_p} dt \wedge dV^{-1} \,, \\ V &= 1 + \frac{Q_1 L_p}{|\vec{x} - \vec{x}_1|} + \frac{Q_2 L_p}{|\vec{x} - \vec{x}_2|} \,. \end{aligned} \tag{3.1}$$

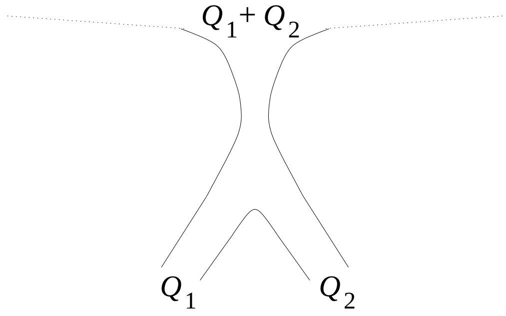

Figure 2: A spatial cross-section of the metric (3.1). There is an asymptotically Minkowskian region and a single charge Q_1+Q_2 throat region which divides into two throats of charges Q_1 and Q_2. In the $M_p \to \infty$ limit (3.3) the throat becomes infinitely long and the Minkowski region decouples. The splitting of the throat into two pieces survives this limit.

Defining

$$\vec{U} = \frac{\vec{x}}{L_p^2} \,, \qquad \vec{U}_1 = \frac{\vec{x}_1}{L_p^2} \,, \qquad \vec{U}_2 = \frac{\vec{x}_2}{L_p^2} \,, \tag{3.2}$$

and taking $L_p \to 0$, the near-horizon metric becomes

$$\begin{aligned} \frac{ds^2}{L_p^2} &= -V^{-2}dt^2 + V^2 d\vec{x}^2, \\ *F &= dt \wedge dV^{-1} \,, \\ V &= \frac{Q_1}{|\vec{U} - \vec{U}_1|} + \frac{Q_2}{|\vec{U} - \vec{U}_2|} \,. \end{aligned} \tag{3.3}$$

The difference $\vec{U}_{12} = \vec{U}_1 - \vec{U}_2$ is a collective coordinate for the solution. The effective action for small variations of this coordinate is easily obtained by scaling the known result [13]

$$S_{12} = \frac{1}{2}(Q_1^3 Q_2 + Q_1 Q_2^3) \int dt \frac{(\partial_t \vec{U}_{12})^2}{|\vec{U}_{12}|^3} \,, \tag{3.4}$$

and is finite for $L_p \to 0$. This geometry is locally flat with a conical singularity. It has the perhaps counterintuitive feature that the volume of the moduli space for widely separated black holes is very small, while that of nearly coincident black holes is divergent. The region corresponding to widely separated black holes is the point $|\vec{U}_{12}| \to \infty$ at the apex of the cone. Nearby black holes occupy the locally asymptotically flat region $|\vec{U}_{12}| \to 0$. It is possible that black hole entropy can be understood as the zero-energy states at the boundary of this moduli space.

Naively this divergent volume leads to an infinite number of arbitrarily low-lying excitations of near-coincident black holes. A similar divergence appears in counting the low-energy modes of a free scalar field in the vicinity of a black hole horizon because of arbitrarily large near-horizon redshifts. Presumably higher order corrections regulate the divergence in both cases, but we do not understand how this comes about.

3.2 The zerobrane limit

It is instructive to consider the case $Q_1 \ll Q_2$. One can then view the charge Q_1 black hole as a charged BPS zerobrane in $AdS_2 \times S^2$ with a constant electric field. The coupled action for such a zerobrane in AdS_2 is

$$S_2 = \frac{1}{4}\int d^2x\sqrt{-g}\left\{e^{-2\phi}\left[R - F^2 + 2(\nabla\phi)^2\right] + \frac{2}{L_p^2}\right\} + \\ + \frac{1}{2}\oint dx\sqrt{h}e^{-2\phi}K + \frac{Q_1}{L_p}\int A - \frac{Q_1}{L_p}\int ds\,. \tag{3.5}$$

The scalar ϕ measures the size of the S^2 and the gauge field strength $F = dA$ is such that $e^{-2\phi}F$ is of order Q_2. The metric on the boundary of AdS_2 is h and K is the extrinsic curvature of the boundary of AdS. The last two terms are the worldline action of the zerobrane.

Taking (3.3) and averaging $\vec{x}_1$ over the two sphere we get

$$\begin{aligned} \frac{ds^2}{L_p^2} &= -\frac{U^2}{Q_2^2}\frac{1}{h(U)^2}dt^2 + \frac{Q_2^2}{U^2}h(U)^2dU^2\,, \\ e^{-\phi} &= Q_2 h(U)\,, \\ \frac{A}{L_p} &= \frac{U}{Q_2}\frac{1}{h(U)}dt\,, \end{aligned} \tag{3.6}$$

where

$$h(U) = 1 + \frac{Q_1}{Q_2}\left(\Theta(U-a) + \frac{U}{a}\Theta(a-U)\right), \tag{3.7}$$

where $\Theta(U)$ is the Heaviside step function. This represents a spherical distribution of zerobranes of total charge Q_1 hovering a fixed distance from the horizon. The zerobrane worldline is at $U = a$, where a is the collective coordinate.

The solution (3.6) covers only the region outside the horizon. The analytic extension of this metric is provided by the Eddington-Finkelstein coordinates[4]

$$d\bar{t} = dt - \left(1 - \frac{Q_2^2}{U^2} h(U)^2\right) dU \,. \tag{3.9}$$

In these coordinates the solution is

$$\begin{aligned} \frac{ds^2}{L_p^2} &= -\frac{U^2}{Q_2^2}\frac{1}{h(U)^2} d\bar{t}^2 + 2\left(1 - \frac{U^2}{Q_2^2}\frac{1}{h(U)^2}\right) d\bar{t}dU + \left(2 - \frac{U^2}{Q_2^2}\frac{1}{h(U)^2}\right) dU^2 \,, \\ e^{-\phi} &= Q_2 h(U) \,, \\ \frac{A}{L_p} &= \frac{U}{Q_2}\frac{1}{h(U)} d\bar{t} \,. \end{aligned} \tag{3.10}$$

Note that this is now regular at $U = 0$, so we have extended the solution to $U < 0$. However, at $U = -Q_2 a/Q_1$, there is a singularity in the metric. This is an essential singularity of the solution; not only does the scalar curvature diverge at this point, but $e^{-\phi}$, which is the size of the internal S^2, is degenerating there (from (3.7)). This is an important contradistinction with pure $AdS_2 \times S^2$.

Empty $AdS_2 \times S^2$ has an inner and an outer boundary, both of which are nonsingular. We have just seen that (3.10) has the feature that the inner boundary is singular in our semiclassical description (see figure 3). This suggests that the dual one-dimensional conformal field theory lives on the outer boundary alone, while some appropriate boundary conditions must be found to define the dynamics of the inner boundary.

Another way to understand the appearance of a singularity in (3.10) is to notice that for $U < a$ the harmonic function appearing in the metric (3.6) is of the form $h(U)/U \sim 1/U + const$ so it is of the same form as that of the extremal Reissner-Nordström black hole before the near-horizon limit is taken.

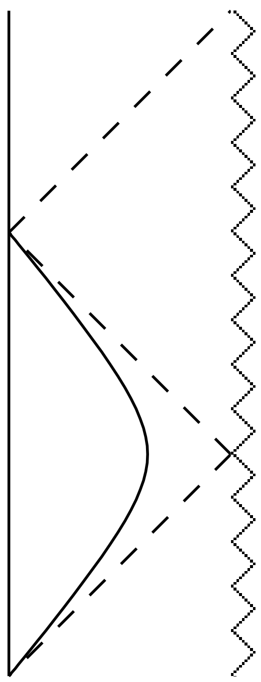

Figure 3: The Penrose diagram for the solution (3.10). The curved path is the zero-brane worldline $U = a$, and the dashed line is the horizon $U = 0$.

3.3 Charged geodesics

In the limit in which the back reaction of the zerobrane on the $AdS_2 \times S^2$ geometry is neglected, it obeys a charged geodesic equation. This equation can be solved in general. We consider first the Euclidean case which is relevant for the discussion of instantons in section 4.

[4]The finite coordinate transformation is

$$\bar{t} - \bar{t}_0 = t - U - \frac{Q_2^2}{U} h(U)^2 - 2\left(\frac{Q_1^2}{a^2}(a - U) - \frac{Q_1 Q_2}{a} \ln \frac{U}{a}\right) \Theta(a - U) \,. \tag{3.8}$$

It is easiest to calculate the zerobrane trajectories in Poincare coordinates

$$ds^2 = \frac{dt^2 + dy^2}{y^2}\,. \tag{3.11}$$

where the worldline action in (3.5) is

$$S = m \int dt \frac{\sqrt{1 + \left(\frac{dy}{dt}\right)^2} - 1}{y}\,. \tag{3.12}$$

The classical solutions to (3.12) are circles of arbitrary radius which are tangent to the boundary

$$(t - t_0)^2 + (y - a)^2 = a^2 \tag{3.13}$$

as shown in fig. 4a. There is an additional solution

$$y = y_0\,. \tag{3.14}$$

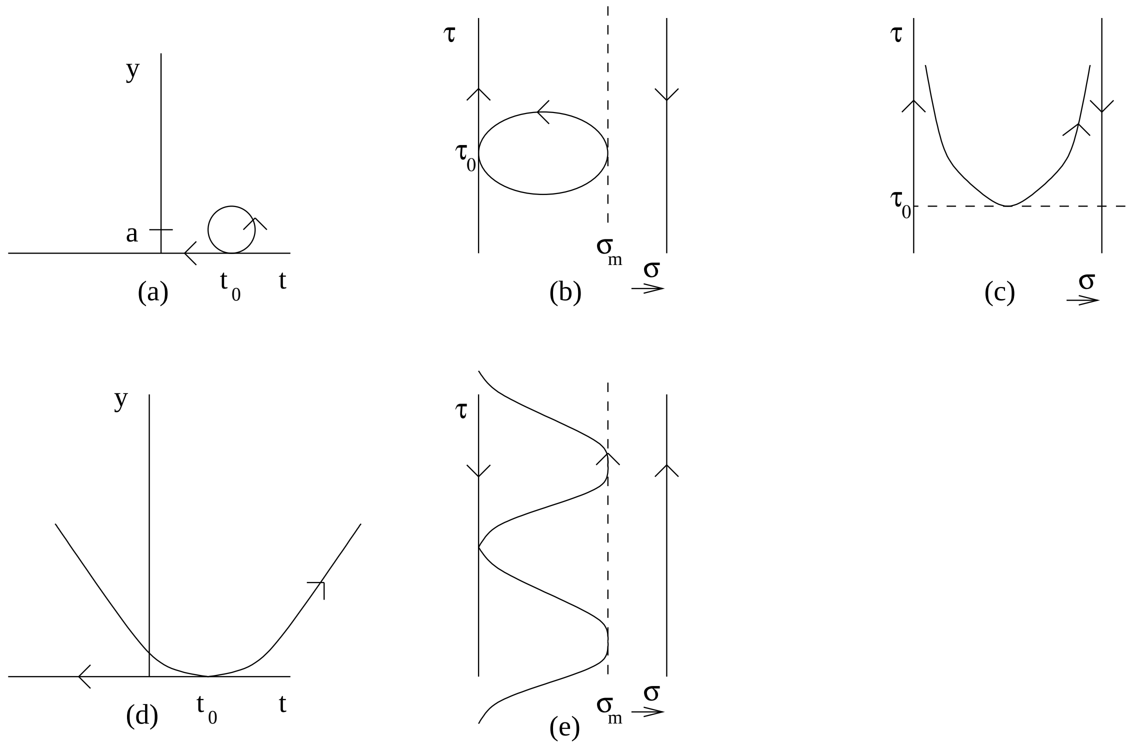

Figure 4: Trajectories of BPS charged particles in AdS_2 with Euclidean and Lorentzian signatures in various coordinate systems. The orientation of the arrows indicate the charges of the particles and the charges of the boundaries. *a*) Euclidean trajectory in Poincare coordinates. *b*) Euclidean trajectory in the strip coordinates. *c*) Zero energy Euclidean trajectory in strip coordinates—this will be relevant for tunneling. *d*) Lorentzian trajectory in Poincare coordinates. *e*) Lorentzian trajectory in (global) strip coordinates.

These solutions are transformed into one another by the action of $SL(2,\mathbb{R})$. In the strip coordinates

$$ds^2 = \frac{d\tau^2 + d\sigma^2}{\sin^2\sigma}\,, \tag{3.15}$$

the action becomes

$$S = m\int d\tau \frac{\sqrt{1+\left(\frac{d\sigma}{d\tau}\right)^2} - \cos\sigma}{\sin\sigma}\,. \tag{3.16}$$

In this case the solutions are generically closed curves tangent to the boundary. For $t_0 > 0$, these are tangent to $\sigma = 0$, and can be written

$$\cosh(\tau - \tau_0) = \frac{\cos(\sigma_m/2 - \sigma)}{\cos\sigma_m/2}\,, \tag{3.17}$$

as shown in fig. 4b. The $t_0 < 0$ solutions, tangent to $\sigma = \pi$, are mirror images of fig. 4b, including orientation. The additional solution is (see fig. 4c)

$$e^{-(\tau-\tau_0)} = \sin\sigma\,. \tag{3.18}$$

This trajectory has zero Euclidean energy and is relevant to vacuum tunneling. The time-reverse of (3.18) is also a geodesic, corresponding to (3.13) with $t_0 = 0$.

Lorentzian trajectories can be obtained by Wick rotation. In Poincare coordinates they are (see fig. 4d)

$$-(t-t_0)^2 + (y+a)^2 = a^2\,, \tag{3.19}$$

Similarly in the strip coordinates we get (see fig. 4e)

$$\cos(\tau - \tau_0) = \frac{\sin(\sigma_m/2 - \sigma)}{\sin(\sigma_m/2)} \tag{3.20}$$

and its mirror image. Note that the particle gets to the boundary in finite global time.

3.4 Multi-black hole configurations

The near-horizon two-black hole geometry (3.3) can be generalized to n black holes simply by replacing V in (3.3) with

$$V = \sum_{i=1}^{n} \frac{Q_i}{|\vec{U} - \vec{U}_i|}\,. \tag{3.21}$$

The resulting geometry has n $AdS_2 \times S^2$ regions near the n horizons $\vec{x} = \vec{x}_i$ as well as an asymptotic one at large $\vec{x}$. Timelike singularities lie behind each of the n horizons. The $3n$-dimensional effective action deduced from [13] is

$$S = \frac{3}{16\pi}\int dt \sum_{i,j,k,l} Q_iQ_jQ_kQ_l |\partial_t\vec{U}_i - \partial_t\vec{U}_j|^2 \int d^3U \frac{(\vec{U}-\vec{U}_i)\cdot(\vec{U}-\vec{U}_j)}{|\vec{U}-\vec{U}_i|^3|\vec{U}-\vec{U}_j|^3|\vec{U}-\vec{U}_k||\vec{U}-\vec{U}_l|}. \tag{3.22}$$

Black holes encountered in string theory tend to involve more than just a single type of charge. This leads to non-trivial generalizations of (3.22). In five dimensions the black holes considered in [14] have three charges Q_1, Q_5 and n. The near-horizon geometry of a single black hole is $AdS_2 \times S^3$. The moduli space geometry was studied in [15]. The near-horizon effective action for p black holes is

$$S = \frac{1}{4}\int dt \sum_{i\neq j}^{p}\sum_{k}^{p}\left(Q_{1i}Q_{5j}n_k + Q_{1i}n_jQ_{5k} + Q_{5i}n_jQ_{1k}\right)\left|\partial_t\vec{U}_i - \partial_t\vec{U}_j\right|^2 \times \\ \times\left[\frac{1}{|\vec{U}_i-\vec{U}_j|^2|\vec{U}_i-\vec{U}_k|^2} + \frac{1}{|\vec{U}_i-\vec{U}_j|^2|\vec{U}_j-\vec{U}_k|^2} - \frac{1}{|\vec{U}_i-\vec{U}_k|^2|\vec{U}_j-\vec{U}_k|^2}\right], \tag{3.23}$$

where $U = r\sqrt{\frac{RV}{g^2\alpha'^4}}$. In the sum, there are divergent terms when $k = i$ or j, but they cancel. Specializing to the case of two black holes, we have

$$S = \frac{1}{2}\int dt \frac{\Gamma_3}{|\vec{U}_1 - \vec{U}_2|^4}\left|\partial_t\vec{U}_1 - \partial_t\vec{U}_2\right|^2 , \tag{3.24}$$

where

$$\Gamma_3 = Q_{11}Q_{52}n_2 + Q_{51}n_2Q_{12} + n_1Q_{12}Q_{52} + Q_{12}Q_{51}n_1 + Q_{52}n_1Q_{11} + n_2Q_{11}Q_{51} . \tag{3.25}$$

The four-dimensional multi-charge expression was found in [16]. Note that, like the single-charge four dimensional case (3.4), the moduli space is locally flat. In fact, the two black hole case (3.24) is exactly $\mathbb{R}^4$; $U = \infty$ is an ordinary point, and the 3-sphere at $U = 0$ is asymptotic infinity.

3.5 The dual CFT picture

Up to this point our discussion has been largely in the framework of semiclassical quantum gravity, and has not significantly involved string theory. In this section we discuss the AdS_2 trees in terms of the dual CFT on the boundary.

In the case of AdS_5 analogous multi-center gravity solutions (and D3 branes in AdS) correspond to Coulomb branches in the field theory where the scalar fields in the vector multiplets have expectation values and the gauge symmetry is broken to subgroups. Fields in a 0+1–dimensional quantum mechanical system do not have well defined expectation values; they fluctuate. The AdS_2 trees correspond to different classical vacua of a quantum mechanical theory and we expect that the system moves continuously among them.

It seems natural to ask whether these trees correspond in any sense to the Coulomb or Higgs branches of the theory. In the classical $g \to 0$ limit of a D-brane system these two branches are very different. Quantum mechanically the system fluctuates and explores the whole moduli space. It has been argued in similar contexts in [17, 18] that the two branches decouple in the low energy limit and that the system, if initially in the Higgs branch, explores only the Higgs branch and does not wander on to the Coulomb branch. The tunneling processes discussed below seem to suggest that the branches are not decoupled. However this is not the end of the story, since it could be that branes in AdS_2 could correspond to something analogous to "small" instantons or similar configurations in the Higgs branch. For example in the AdS_5 description of Yang Mills field theory on S^4 a Yang Mills instanton — which corresponds to the Higgs branch of the D(-1) brane gauge theory — is a D(-1) brane moving in AdS_5 [19] . This implies that D(-1) branes in the near-horizon geometry are already dissolved in the D3-brane field theory and that we should interpret their positions as sizes. This analogy is imperfect because there is no Coulomb branch at all for the S^4 Yang-Mills theory. Nevertheless it suggests the possibility that a brane in AdS_2 or an AdS_2 tree could be corners of the Higgs branch near the point where the Coulomb branches meet. This view is corroborated by the observation that the volume of the moduli space for a 0-brane — as discussed above — in AdS_2 is finite near $U_{12} = \infty$.

4. AdS fragmentation

In the previous section we discussed configurations in which a single charge $Q_1 + Q_2$ $AdS_2 \times S^2$ can branch into two $AdS_2 \times S^2$ spaces with charges Q_1 and Q_2 as one moves spatially from infinity towards the horizon. As can be seen from the analysis of geodesics in section 3.3, this branching point can actually reach the boundary of $AdS_2 \times S^2$ in finite global time. In principle the geometry could then fragment into two completely separate $AdS_2 \times S^2$ universes with charges Q_1 and Q_2. Whether or not this actually happens depends on the boundary conditions at the $AdS_2 \times S^2$ boundary.

In this section we will consider some examples in which fragmentation of a single AdS universe into several smaller AdS universe does occur, for both AdS_2 as well as higher dimensional examples. The processes we consider are tunneling processes mediated by topology-changing instantons. These or closely related instantons were considered in a different context in [7, 8]. The first two subsections consider processes in which an initial AdS space fragments into one macroscopic and one microscopic component. The microscopic component is described by a brane. We then turn to the case (analyzed by Brill [9]) in which it splits into two macroscopic AdS universes.

4.1 The non-supersymmetric case

Consider AdS_D for general D endowed with a constant antisymmetric D-form field strength. In flat space, a constant antisymmetric D-form field strength leads to $(D-2)$-brane creation for any variety of $(D-2)$-brane that is charged under the D-form field strength [8] . In the case of $D = 2$ the field strength is a two-form and this reduces to the well known Schwinger pair production of 0-brane anti-0-brane pairs. The Schwinger process is described by an instanton in which the charged particle moves in a circular trajectory in the electromagnetic field. The vacuum decay rate is proportional to e^{-S_e} where S_e is the action of this Euclidean solution. The configuration to which the vacuum decays — namely a 0-brane anti-0-brane pair — is found by cutting the instanton in half at the moment of time symmetry (say $\tau = 0$). The branes subsequently accelerate off to infinity. For general D the analogous Euclidean solution is a $D-1$ sphere. Cutting it in half we get at $\tau = 0$ a $D-2$ sphere. In the subsequent Lorentzian evolution the sphere expands due to the force exerted by the D-form field strength. In flat space this process of brane creation screens the D-form field strength. In AdS_D $(D-2)$-branes can also be created in this fashion [7] .

We will here describe these instantons in the test-brane approximation, where we neglect the charge of the brane compared with the total flux of F_D in AdS_D. Let us write the metric of AdS_D as

$$ds^2 = R^2 \left(\cosh^2\!\rho \; d\tau^2 + d\rho^2 + \sinh^2\!\rho \; d\Omega^2_{D-2}\right), \tag{4.1}$$

where R is the anti-de Sitter radius. Then the action of a spherically symmetric brane coupled to the D-form field strength is

$$S = TR^{D-1}\Omega_{D-2} \int d\tau \left[\sinh^{D-2}\rho \sqrt{\cosh^2\rho + \left(\frac{d\rho}{d\tau}\right)^2} - q\sinh^{D-1}\rho\right], \tag{4.2}$$

where T is the brane tension, q is the ratio of the charge of the brane to its tension and

$$\Omega_{D-2} = \frac{2\pi^{\frac{(D-1)}{2}}}{\Gamma(\frac{D-1}{2})} \tag{4.3}$$

is the volume of a unit $D-2$ sphere. In the BPS case the forces balance so $q = 1$. In a supersymmetric theory the BPS bound implies that $q \leq 1$ for all possible branes. In a non-supersymmetric context $q > 1$ is possible (for example the electron). In the next subsection an example is given in string theory.

Now we turn to solutions of the brane action (4.2) for $q > 1$. Since the action (4.2) is independent of time, Euclidean energy is conserved. For a spherically symmetric and compact surface, this energy is zero. Energy conservation then implies

$$\frac{\cosh^2\rho}{\sqrt{\cosh^2\rho + \dot{\rho}^2}} - q\sinh\rho = 0\,, \tag{4.4}$$

which is independent of D. The solution of this equation is

$$\cosh\rho = \frac{\cosh\rho_{max}}{\cosh\tau}\,, \tag{4.5}$$

where $\tanh \rho_{max} = 1/q$. Equation (4.5) describes a closed $D-1$ surface with maximum radius ρ_{max}. The action of this instanton is

$$S_{inst} = \frac{2TR^{D-1}\Omega_{D-2}}{\sinh \rho_{max}} \int_0^{\rho_{max}} d\rho \frac{\sinh^{D-2}\rho\sqrt{\sinh^2\rho_{max} - \sinh^2\rho}}{\cosh\rho}$$
$$= \frac{\pi^{\frac{D}{2}} T R^{D-1}}{\Gamma(\frac{D+2}{2})} \sinh^{D-1}\rho_{max} F\left(1, \frac{D-1}{2}; \frac{D+2}{2}; -\sinh^2\rho_{max}\right). \quad (4.6)$$

At $\tau = 0$ one can match (4.5) to the Lorentzian solution

$$\cosh\rho = \frac{\cosh\rho_{max}}{\cos\tau}, \quad (4.7)$$

which describes the post-tunneling evolution. Note that the brane gets to the boundary (at $\rho = \infty$) in finite time (at $\tau = \pi/2$) (see fig. 5).

If $q < 1$ the Euclidean solution is

$$\cosh\rho = \frac{\sinh\tau_{max}}{\sinh\tau}, \qquad \tanh\tau_{max} \equiv q. \quad (4.8)$$

It does not describe a tunneling process because the solution does not have a moment of time symmetry and also its action is infinite. This is expected since tunneling is forbidden by energy conservation for $q < 1$.

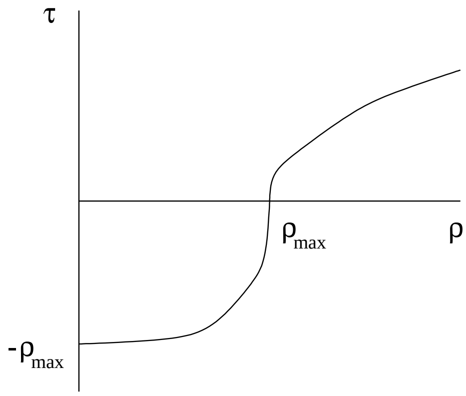

Figure 5: The Euclidean instanton (4.5) for $\tau < 0$ matches onto the Lorentzian solution (4.7) at $\tau = 0$.

4.2 An example of a brane with charge greater than tension

In this section we will give an example of a non-supersymmetric $AdS_3 \times S^3 \times K3$ compactification which is unstable to fragmentation. Consider type IIB on $K3$. The six dimensional theory has an $SO(5,21)$ multiplet of strings, coming from branes wrapped on various cycles of $K3$. A string is characterized by a charge vector q^I transforming in the vector under $SO(5,21)$. If $q^2 > 0$ the string is BPS and the near-horizon geometry is $AdS_3 \times S^3 \times K3$. If $q^2 < 0$ the string will not be BPS, but nevertheless there is a supergravity solution and its near-horizon geometry is again $AdS_3 \times S^3 \times K3$ [20]. A simple example is the following. Take a set of Q_5 D5-branes wrapped on $K3$ which leads to a string in six dimensions. We can add D1-branes to this system. Supersymmetry is preserved only if the D1-branes have the right charge — we choose it to be positive. The BPS bound for the tension of the string is $T \sim |Q_5 V_4 + Q_1|/g$ where V_4 is the volume of $K3$ in string units. When we wrap a D5-brane on $K3$ there is one unit of negative D1-brane charge induced on the brane [21]. So there are BPS strings with charges $(q_5, q_1) = (1, -1)$ whose tension is $T_{(1,-1)} = (V_4 - 1)/g$ (we are assuming $V_4 > 1$). Supersymmetric black strings will have charges $Q_1 \geq 0$. Consider a black string whose charge is $Q_1 < 0$.

The bosonic part of the supergravity equations have a structure which is not very sensitive to the relative sign of Q_1. Actually the only change in the solution is the sign in the electric part of the B field. In particular, the volume of the $K3$ at the horizon, (encoded in the dilaton), will be $V = |Q_1|/Q_5$ so we will take $|Q_1| > Q_5$ so that the above formula for the tension of the $(1, -1)$ string is correct.

We can also study small fluctuations around this solutions and check that there is no tachyon producing an instability (i.e. all tachyons obey the Breitenlohner-Freedman bound $m^2 \geq -1$). The basic reason is that the equations involve a coset space $SO(5, 21)/SO(5) \times SO(21)$ and self-dual field strengths transforming in the **5** of $SO(5)$ and antiself-dual field strengths transforming in the **21**. In the case that $Q_1 > 0$ only the self-dual field strength is non-zero while if $Q_1 < 0$ only the antiself-dual part is non-zero in the near horizon region. The bosonic equations are symmetric under the exchange of self-dual and antiself-dual fields together with the change of the $SO(5)$ and $SO(21)$ pieces of the coset. So if for $Q_1 > 0$ we had 21 fixed scalars and a similar number of "good" tachyons now we will have 5 fixed scalars and a similar number of "good" tachyons, etc. In the negative Q_1 case the string with charges $(q_5, q_1) = (1, -1)$ will feel a repulsive force, since the only difference with the supersymmetric case (in which the forces balance) is that the onebrane electric force is repulsive rather than attractive. So this string will have $q > 1$ in the notation of the previous section (more precisely $q = \frac{|Q_1|+Q_5}{|Q_1|-Q_5}$). This implies that the black hole would lose its charge by emitting these $q > 1$ branes.

It is natural to ask whether other non-supersymmetric AdS spaces would have similar instabilities. These kind of instabilities would generically occur for AdS_2 cases; a well known example is an extremal electrically charged black hole in our universe, which discharges by emitting electrons (which have $q \sim 2 \times 10^{21}$). It can be seen that if we have an $AdS_5 \times M^5$ compactification where M^5 is an Einstein manifold, then the gravity equation of motion implies that a three-brane moving in that geometry will have $q = 1$ regardless of whether supersymmetry is broken or not. There could be, however, other branes with $q > 1$ if the Einstein manifold has some small cycles on which one can wrap branes etc. In cases where one has a "warped" geometry [22], i.e. a solution where the radius of AdS_5 depends on the coordinates on M^5 then the threebrane could have $q > 1$. An example is the compactification that has $SO(5)$ symmetry that arises as an unstable IR "fixed point" after perturbing the $\mathcal{N} = 4$ theory by a relevant operator [23].

The nature of the tunneling process is illuminated by consideration of the energy of a (momentarily) static spherical brane at radius ρ. This is not an equilibrium configuration. This energy has a positive contribution from the mass of the brane and a negative contribution from the electric potential:

$$E(\rho) = TR^{D-2}\Omega_{D-2}\left[\sinh^{D-2}\rho\cosh\rho - q\sinh^{D-1}\rho\right]. \tag{4.9}$$

We see that if $q > 1$ then $E(\rho) \to -\infty$ as $\rho \to \infty$. So the system decays in order to reach this lower energy configuration. If $q = 1$ then we see that $E(\rho) \to \{\infty, const, 0\}$ for the $\{D > 3, D = 3, D = 2\}$ cases as $\rho \to \infty$. This is consistent with the fact that only for $D = 2$ can we have tunneling in the $q = 1$ case.

It is interesting to note that in the AdS_3 case the constant value of the energy is related to the change in the central charge of the system when we remove a brane. This is anticipated from the dual NS-sector CFT description in which the ground state energy is proportional to the central charge. More precisely, if we have a D5-D1 brane system then the change in energy is $Q_1/2$ if we remove a fivebrane and $Q_5/2$ if we remove a onebrane. This further implies that if we have enough energy above the ground state in AdS_3 then the system can decay by emitting branes. In the case with NS charges (NS fivebranes and fundamental strings) this decay mode might be related to the negative norm states for the $SL(2, \mathbb{R})_{Q_5}$ WZW model arising when the square of the mass of the state is of the order of Q_5.

4.3 The supersymmetric case

In this subsection we consider the supersymmetric case $q = 1$. The solution to the geodesic equation for $q = 1$ is

$$e^{\tau} = \cosh \rho \tag{4.10}$$

as discussed previously in (3.18). Unlike the $q < 1$ case (4.10) is not a "bounce" solution. It does not have a moment of time symmetry and there is no negative mode indicating an instability. Rather it represents tunneling between two degenerate vacua. At $\tau = -\infty$ the instanton is asymptotic to charge $(Q_1 + Q_2)$ AdS_2, while at $\tau = +\infty$ one has charge Q_2 AdS_2 plus a charge Q_1 brane in the boundary.

For $D > 2$ the instanton action is infinite so the tunneling does not actually occur. For $D = 2$ it takes the finite value

$$S_{instanton} = \pi Q_1 Q_2 \,. \tag{4.11}$$

Comparing with (2.3) we see that this can be written

$$S_{instanton} = -\frac{1}{2}\Delta S_{BH} \,, \tag{4.12}$$

where ΔS_{BH} is the difference in the Bekenstein-Hawking entropy of the initial and final states. In a description which microscopically accounts for the Bekenstein-Hawking entropy, twice the factor (4.12) would arise in transition *probabilities* from averaging over initial and summing over final states. This agrees with the fact that the instanton (4.12) gives transition *amplitudes*. Apparently the instanton mysteriously knows the number of microstates. Previous examples of instantons counting microstates in this fashion can be found in [24, 9].

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All supersymmetric instantons will have fermion zero modes which we have not analyzed in detail. This means that the transitions will be accompanied by a change in fermion number and/or spacetime momentum,[5] and will not shift the ground state energy.

4.4 The Brill instanton

A tunneling process in which an initial $AdS_2 \times S^2$ universe fragments into two final $AdS_2 \times S^2$ universes should be described by a smooth instanton with one initial and two final $AdS_2 \times S^2$ boundaries. Such an instanton was discovered by Brill [9], whose work we review in this subsection. In the limit in which one of the final universes is small and can be treated like a brane, this instanton reduces to (4.10) of the previous subsection.

The Euclidean action for the Einstein-Maxwell theory is

$$S = -\frac{1}{16\pi}\int d^4x\sqrt{g}(R - F^2) - \frac{1}{8\pi}\oint d^3x\sqrt{h}K\,, \tag{4.13}$$

where K and h are the trace of the extrinsic curvature and the induced metric on the boundary and we have set Newton's constant to one. This has the family of solutions

$$\begin{aligned} ds^2 &= V^2 d\vec{x}^2 + V^{-2}dw^2\,, \\ *F &= -dw \wedge dV^{-1}\,, \\ \vec{\nabla}^2 V(\vec{x}) &= 0\,, \end{aligned} \tag{4.14}$$

where $\vec{\nabla}^2$ is the Laplacian on flat $\mathbb{R}^3$. The special case

$$V = \frac{Q_0}{|\vec{x}|} \tag{4.15}$$

corresponds to the Euclidean $AdS_2 \times S^2$ Robinson-Bertotti universe with magnetic charge Q_0 on the S^2, and AdS_2 cosmological constant $\frac{2}{Q_0}$. The Brill instanton is[6]

$$V = \frac{Q_1}{|\vec{x} - \vec{x}_1|} + \frac{Q_2}{|\vec{x} - \vec{x}_2|}\,. \tag{4.16}$$

For $\vec{x} \to (\vec{x}_1, \vec{x}_2, \infty)$ the metric given by (4.16) approaches the $AdS_2 \times S^2$ metric (4.15) with charge $Q = (Q_1, Q_2, Q_0 = Q_1 + Q_2)$.

We wish to interpret this as a semiclassical contribution to the tunneling of an initial charge $Q_0 = Q_1 + Q_2$ $AdS_2 \times S^2$ spacetime to final charge Q_1 and Q_2

[5] This momentum refers to the original asymptotically flat region. In the AdS context it is conjugate to zero modes of singleton fields.

[6] This metric is obtained by analytic continuation $t \to iw$ of (3.3), but we will not interpret w as Euclidean time.

spacetimes. In order to do so we need to identify one initial surface Σ_0 and two final surfaces Σ_1 and Σ_2 with topologies $\mathbb{R} \times S^2$ corresponding to spatial slices of $AdS_2 \times S^2$ spacetimes with the appropriate charges. The metrics on the surfaces should agree with those of the corresponding slices of $AdS_2 \times S^2$. The extrinsic curvatures should vanish so that the continuation back to Lorentzian signature gives real initial data.[7]

Let us introduce the following variable

$$y = \left(\frac{Q_1}{\sqrt{|\vec{x} - \vec{x}_1|}} + \frac{Q_2}{\sqrt{|\vec{x} - \vec{x}_2|}} \right)^2 , \tag{4.17}$$

and then define the variables τ, σ through

$$w + iy = e^{\tau + i\sigma} , \tag{4.18}$$

where $0 \leq \sigma \leq \pi$. We see that for $\tau \to -\infty$ we have the single charge Q_0 AdS space in the (Euclidean) strip coordinates and for large τ we have two AdS_2 spaces of charges Q_1 and Q_2 which actually meet at $\sigma \to 0, \pi$ to form again a charge Q_0 AdS_2 space (the change of variables (4.17) is, in a sense, double valued). The point σ where they meet goes to the boundary ($\sigma = 0, \pi$) when $\tau \to +\infty$ so that in the limit we really have two disconnected AdS_2 spaces. We need to regulate the spatial extent of AdS_2 (the σ coordinate) and also the temporal extent. This could be achieved by taking cutoffs $\epsilon < \sigma < \pi - \epsilon$ and $-T < \tau < T$. Notice that if we take first a finite ϵ, then the two final AdS_2 regions become disconnected at finite T.

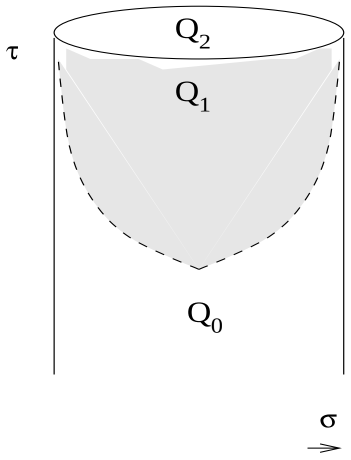

Figure 6: The Brill instanton. We start with an charge $Q_1 + Q_2$ $AdS_2 \times S^2$ space which splits into two $AdS_2 \times S^2$ spaces of charges Q_1 and Q_2. For finite Euclidean time they still look like a single charge AdS space close to the boundary. Only at infinite time do the two AdS spaces separate.

The tunneling amplitude is proportional to the exponential of minus the instanton action. Reducing the action to a surface term and subtracting the action of the vacuum to vacuum instanton, Brill finds an amplitude proportional

$$A_{Q_0 \to Q_1 + Q_2} \sim e^{\frac{1}{2}\Delta S_{BH}} , \tag{4.19}$$

where

$$\frac{1}{2}\Delta S_{BH} = \frac{1}{2}\{S_{BH}(Q_1) + S_{BH}(Q_2) - S_{BH}(Q_1 + Q_2)\} = -\pi Q_1 Q_2 . \tag{4.20}$$

In this expression,

$$S_{BH}(Q) = \pi Q^2 \tag{4.21}$$

is the Bekenstein-Hawking entropy for a charge Q extremal black hole. Squaring the amplitude to get the transition probability one finds that it is proportional to minus

[7]The surfaces given in [9] differ from those described here and do not satisfy this criterion.

the exponential of the entropy decrease, as expected. This result agrees exactly with the result (4.11) computed for $Q_1 \ll Q_2$. Because of the necessity of subtractions, it is not manifestly obvious (although it is expected) that the action will be the same when computed for the initial and final surfaces described above.

An interpretation of the tunneling process which does not refer to the decoupled asymptotically flat region can be given in the context of third-quantized Hilbert space in which states are labeled by the occupation numbers n_i of $AdS_2 \times S^2$ spacetimes with charges Q_i. The Brill instanton corresponds to a nonperturbative correction to the Hamiltonian which changes these occupation numbers. Due to charge conservation there are superselection sectors labeled by the total charge. In the semiclassical approximation considered here, the $AdS_2 \times S^2$ spacetimes are like non-relativistic particles. There is no pair-creation of oppositely charged spacetimes, and Q_i is restricted to be positive.

In general topology-changing processes which change the number of universes are problematic (a review can be found in [25]). Among other reasons, it is difficult to describe such processes by a Hamiltonian because there is in general no canonical way to compare the times of different universes. This problem cannot arise in the present context because it was derived as a limit of a system which included the asymptotically flat region and did have a Hamiltonian. The separate $AdS_2 \times S^2$ universes carry a preferred time with them as a remnant of the asymptotically flat region which once joined them.

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